A mathematical analysis of the influence of wind uncertainty on MTCD efficiency

Jean-Marc Alliot (DSNA/R&D)
Nicolas Durand (DSNA/R&D)

Abstract

A large part of the controller's workload comes from conflict detection and monitoring. The SESAR project aims at giving tools (such as MTCD) to air traffic controller that will lighten this part of their burden and help them to have a more strategic planning activity while letting the computer take into account some of those "housekeeping" tasks.

In this article we explain how the conflict detection task can be analyzed mathematically, and what can be learned from this theoretical study. We will also show that, whatever the quality of MTCD tools, because some uncertainties like wind prediction errors are unavoidable, even a perfect MTCD will always detect more conflict that the actual number of conflicts that really occur.

Understanding conflict detection from a mathematical point of view

Figure 1 shows a classical two aircraft conflict. Aircraft on the lower segment flies at speed v_1, and aircraft on the upper segment flies at speed v_2. The angle of incidence is α:

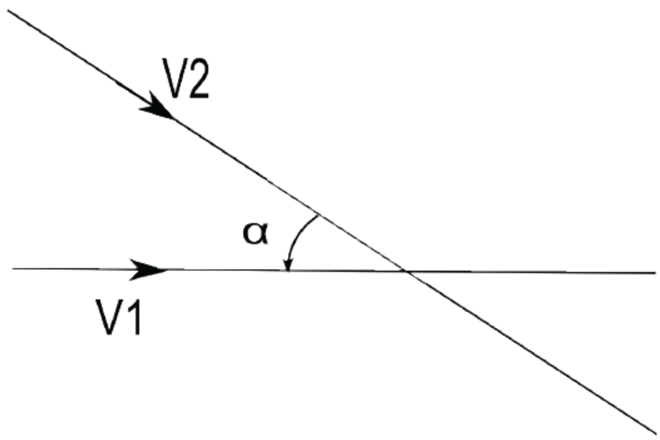


Figure 1

We use the auxiliary variables $r = \frac{v_2}{v_1}$, and D is the separation standard. Let's suppose that we have an aircraft p_1 on the lower segment at a distance l_1 of the crossing point. We want to know which interval on the upper segment will contain conflicting aircraft with this one. Let's assume that an aircraft p_2 is at distance l_2 of the crossing point. Then we have:

$$x_1 = v_1 t - l_1$$

$$y_1 = 0$$

$$x_2 = \cos(\alpha)\,(v_2 t - l_2)$$

$$y_2 = \sin(\alpha)\ (v_2 t - l_2)$$

If the two aircraft are in conflict, there must exist t such that the following inequality is satisfied:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 \leq D^2$$

This is a second degree inequality in t. The inequality will only be satisfied if the equation:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 - D^2 = 0$$

has at least one solution. So the discriminant of the equation:

$$\Delta = D^2(v_1^2 - 2v_1 v_2 \cos(\alpha) + v_2^2) - \sin^2\alpha\,(l_2 v_1 - l_1 v_2)^2$$

must be positive. The discriminant is itself a polynomial of degree 2 in l_2. Thus, the distances l_2 which satisfy $\Delta \geq 0$ belong to a single interval, and the extremal points r_1 and r_2 of the interval are the roots of the equation $\Delta = 0$:

$$r_1 = l_1 \frac{v_2}{v_1} + D \frac{\sqrt{1 + (\frac{v_2}{v_1})^2 - 2(\frac{v_2}{v_1}) \cos\alpha}}{\sin\alpha}$$

$$r_2 = l_1 \frac{v_2}{v_1} - D \frac{\sqrt{1 + (\frac{v_2}{v_1})^2 - 2(\frac{v_2}{v_1}) \cos\alpha}}{\sin\alpha}$$

Let's understand this result. It is just a mathematical way to say that if aircraft $p_1$is at a distance l_1 of the conflict point on the horizontal segment, then all aircraft which are at a distance between r_1 and r_2 on the other segment will be in conflict with p_1.

Let's make a simple numerical application. If aircraft p_1 is flying at 400kts and is at a distance of 60Nm of the conflict point, aircraft p_2 is flying at 380kts, the crossing angle is 45° and the separation standard is 5Nm then we have:

$$r_1 = l_1 \frac{v_2}{v_1} + D \frac{\sqrt{1 + (\frac{v_2}{v_1})^2 - 2(\frac{v_2}{v_1}) \cos\alpha}}{\sin\alpha} = 60 \frac{380}{400} + 5 \frac{\sqrt{1 + (\frac{380}{400})^2 - 2\frac{380}{400} \cos 45}}{\sin 45} = 62$$

$$r_2 = l_1 \frac{v_2}{v_1} - D \frac{\sqrt{1 + (\frac{v_2}{v_1})^2 - 2(\frac{v_2}{v_1}) \cos \alpha}}{\sin \alpha} = 60 \frac{380}{400} - 5 \frac{\sqrt{1 + (\frac{380}{400})^2 - 2 \frac{380}{400} \cos 45}}{\sin 45} = 52$$

So each aircraft $p_2$which is closer to the crossing point than 52Nm will safely pass in front of p_1 and each aircraft $p_2$which is further than 62Nm of the crossing point will safely pass behind p_1. All aircraft inside the [52,62] Nm segment will be in conflict with p_1. Of course all of this makes sense only if there are absolutely no uncertainties on the future trajectories of aircraft.

A very interesting value is the length of the segment, which is the difference between the two roots:

$$L = r_1 - r_2 = 2D \frac{\sqrt{1 + (\frac{v_2}{v_1})^2 - 2(\frac{v_2}{v_1}) \cos \alpha}}{\sin \alpha} = 2 * 5 \frac{\sqrt{1 + (\frac{380}{400})^2 - 2 \frac{380}{400} \cos 45}}{\sin 45} = 10$$

This means that for any aircraft $p_1$we have to monitor a length of 22Nm on the upper segment. Let's now use $r = \frac{v_2}{v_1}$ (the ratio of speeds) and let's express the length of the segment in number of separation standard:

$$\frac{L}{D} = f(r, \alpha) = 2 \frac{\sqrt{1 + r^2 - 2r \cos \alpha}}{\sin \alpha} = 2.1$$

This means that if aircraft are as densely packed as possible on the upper segment (they follow each other at a D=5Nm distance in a "miles in trail" procedure), aircraft $p_1$will be in conflict with roughly 2 aircraft. Of course, if the density of the traffic is half the maximal density, there will only be 1 aircraft in conflict with p_1 on the average, etc. This formula is also interesting to understand the importance of the crossing angle. If α=90°, we have roughly 3 aircraft in conflict with p_1 instead of 2.

We notice a few interesting things:

- The function does not depend on l_1. This means that, if information was perfect and there was no uncertainty of any kind, conflicts could be exactly predicted as soon as we know the position of the aircraft, even if they are very far away from the crossing point. This relies of course on the very strong and quite unrealistic assumption that we know the exact trajectory of each aircraft and that aircraft are going to perfectly "stick" to their trajectories.
- The minimum of $f(r, \alpha)$ is reached for $r = cos(\alpha)$, and we have $f(cos(\alpha), \alpha) = 2$. This is independent of α and perfectly normal: if we project the situation to the state where $p_1$is at the crossing point, all aircraft p_2 at a distance $-D < d < D$ of the crossing point are in conflict with p_1, and if they are exactly separated by one separation standard, there will be exactly two aircraft between $-D$ and D.

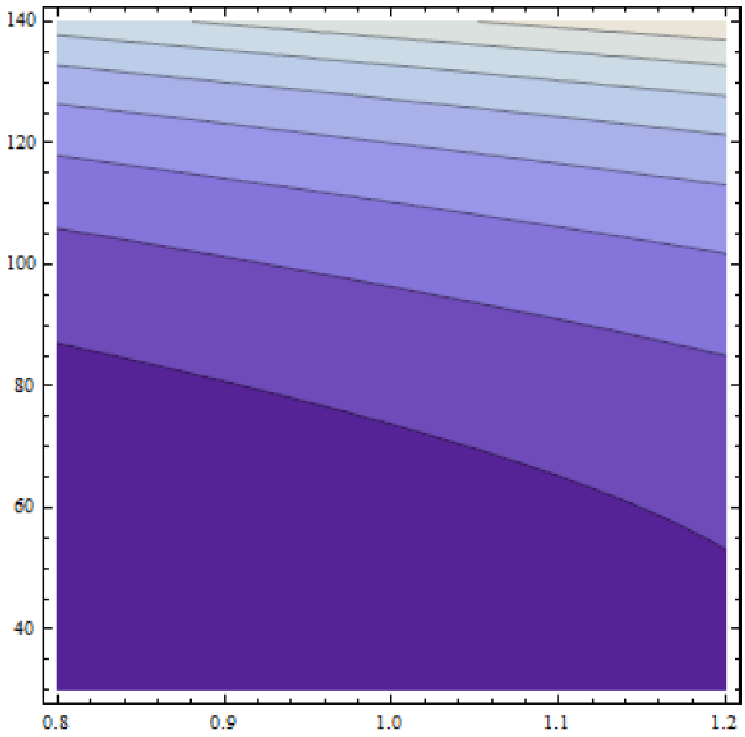


Figure 2

We represent on figure 2 a contour plot of the above function. The horizontal axis is r (the ratio of speeds $r = \frac{v_2}{v_1}$); we use values of r ranging from 0.8 to 1.2. The vertical axis is the angle of incidence (in degrees), from 30 degrees to 140 degrees. We do not represent values above 140 degrees or below 30 degrees, as they exhibit pathological behavior. The darkest part of the contour plot represents segment lengths ranging from 2 to 2.5 separation standard. Each line represents one half separation standard more. For example, if the ratio of speed is 0.8 and the crossing angle is less than 85°, then the length of the upper segment to monitor for a given aircraft p_1 is around 2*5=10Nm. But if the crossing angle is 120°, the length of the segment is slightly more than 3.5*5=17.5Nm. If the crossing angle is still 120° but the ratio of speed is 1.2 then the length to monitor is slightly less than 4.5*5=22.5Nm.

One should naively expect the function to be symmetrical with$f(r,\alpha) = f(1/r,\alpha)$, as the problem looks symmetrical: aircraft on the upper segment should "see" the same number of conflicts with aircraft on the lower segment than aircraft on the lower segment with aircraft on the upper segment. Actually, we have: $f(1/r,\alpha) = r\, f(1/r,\alpha)$. This can be intuitively understood on a simple example.

Let's suppose that aircraft on the upper and the lower segment are equally spaced by exactly one separation standard (maximal rate). Each aircraft on the lower segment will detect roughly $f(r,\alpha)$ conflicts. In a given time period T, $n_1 = T\ x\ (v_1/D)$ aircraft will pass on the lower segment, and the total number of conflicts detected by aircraft on the lower segment will be $n_1\, f(r,\alpha)$. During the same time period T, $n_2 = T\ x\ (v_2/D)$ aircraft on the upper segment will detect $n_2\, f(1/r,\alpha)$ conflicts. These two numbers have to be equal, so: $n_1\, f(r,\alpha) = n_2\, f(1/r,\alpha)$. By replacing:

$$f(r,\alpha) = \frac{T\,\frac{v_2}{D}}{T\,\frac{v_1}{D}} f\left(\frac{1}{r},\alpha\right) = \frac{v_2}{v_1} f\left(\frac{1}{r},\alpha\right) = r\,f(\frac{1}{r},\alpha)$$

The $f(r,\alpha)$ function is an interesting function because it gives a lower bound on the "number" of conflicts that a controller or an air traffic control system would have to solve if everything was perfect, with total information and no uncertainty. We are now going to explain what happens when uncertainties are in our way.

Why is uncertainty important

A large part of the controller's workload comes from trajectory monitoring and conflict detection. Different studies show that only one conflict out of three to five detected and monitored would really result in separation violation. Granger shows in his Ph. D. [Granger1] that, on simulated traffic on a busy day of 1999 in the French airspace above FL 320, when the uncertainties on trajectory predictions increase, the number of necessary maneuvers also increases dramatically. For example, without uncertainty, 972 conflicts occur and 1041 maneuvers are required to solve them. When you consider a 2% uncertainty on ground speed and 5% uncertainty on climbing or descending rate, 2461 maneuvers are necessary to solve the detected conflicts. The number of maneuvers rises to 3881 maneuvers for a 5% uncertainty on ground speed and 15% uncertainty on vertical speed. It reaches 6819 maneuvers with 10% ground speed and 30% vertical speed uncertainties. This phenomenon is well known by controllers who have to monitor and even sometimes solve conflicts that will often never occur: their priority is safety and they have to take into account uncertainty margins the best they can.

Another statistical study [Alliot1] computes on real data simulation in the french airspace the influence of the vertical and ground speed errors on conflict probe. The formula which summarizes this study is:

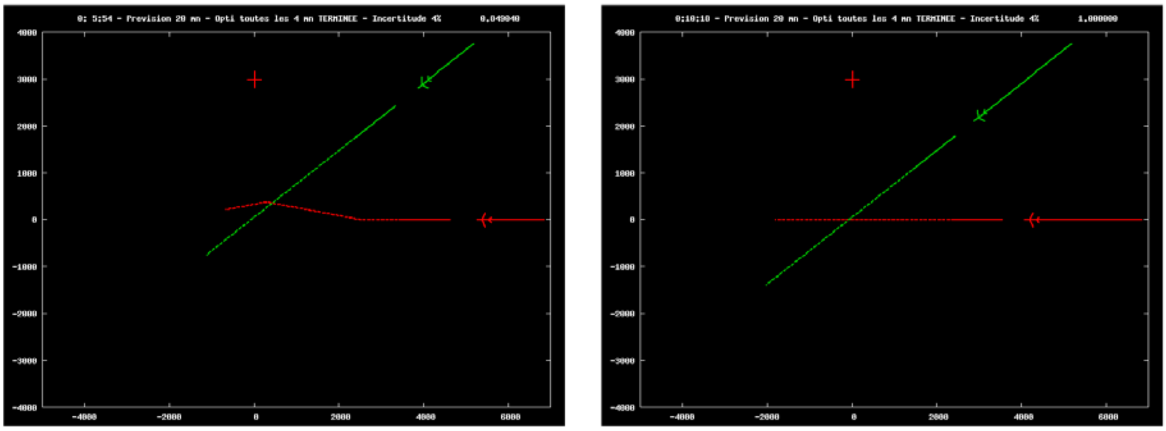
$$N_d/N_0 = 1 + t_w\,(3.5\,e_g + 0.5\,e_V)$$

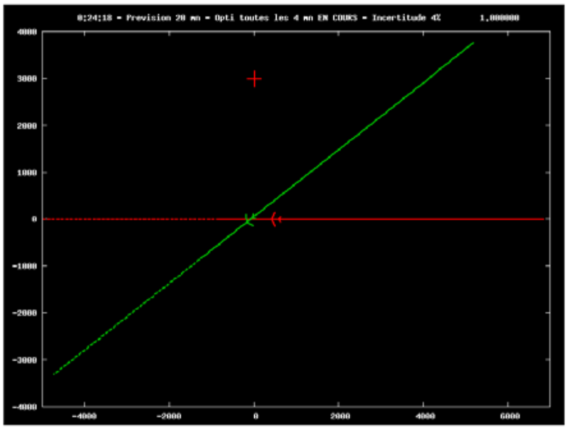
where N_d/N_0 stands for the ratio of conflicts detected over conflicts really happening. t_w is the prediction anticipation, e_g the ground speed error (in percentage) and e_V the vertical speed error (in percentage also).

This is directly the consequence of uncertainties affecting aircraft trajectories, of the incomplete information regarding aircraft speed and intentions, and of human beings inability to handle complex numerical mathematic to compute precisely trajectory predictions. Of course these results are statistical. They depend on the geometry of the airspace monitored, on the traffic density and shape, on the anticipation chosen, and on numerous other factors. In the next part of this paper we are going to develop the underlying mathematical model that explains these results.

We will study in detail in the next section the special problem of wind, which is an unavoidable uncertainty. We are first going to say here a few words about uncertainties which are supposed to be avoidable.

On the following figure, we show a classical example of a detection/resolution process:





1) Up and left: aircraft are far away from the crossing point. Uncertainty is large, the controller detects a potential conflict and gets ready to solve this conflict by turning the red aircraft to the right. However he still has time before the conflict occurs and he decides to wait and monitor the situation.

2) Up and right: the aircraft are closer to the conflict point and uncertainty is much smaller. Now it looks quite clear that aircraft won't be in conflict, and the controller can either monitor the aircraft or rely on an ATC tool to monitor the situation for him.

3) Down and left: aircraft have crossed and the problem is over.

Figure 3

Thus, as stated above, a controller has to monitor and has to plan solving a much larger number of conflicts than the theoretical number found in the above section.

The advocates of MTCD tools claim that it is possible to enhance the efficiency of conflict detection by (a) relying on the computer to build better trajectory prediction and (b) enhance further these predictions by using information downloaded from the aircraft FMS.

Regarding hypothesis (a) a computer is, of course, much more efficient at doing calculations than a human being. Moreover, it is possible, even if we do not have the FMS information available to build correct mathematical models of speed errors and construct MTCD tools from these models. For example, we assume that there is an error about the aircraft future location because of ground and vertical speed prediction uncertainties. Then, an aircraft is represented by a point at the initial time of the conflict detection window.

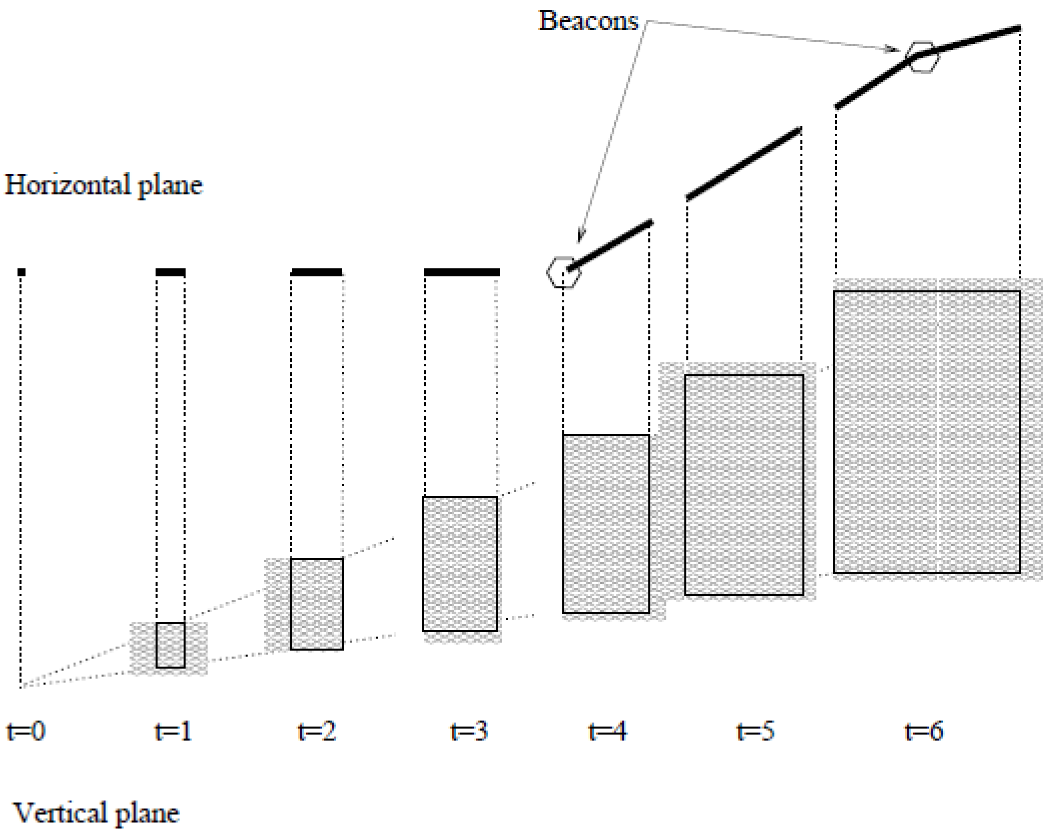


Figure 4

In the horizontal plane, the point becomes a line segment in the uncertainty direction (the speed direction here (see figure 4 above). The first point of the line “flies” at the maximum possible speed, and the last point at the minimum possible speed. These maximal and minimal speeds depend of course on the uncertainty chosen: for 5% uncertainty on ground speed, the first point will fly at a speed of $1.05\,v$ and the last point at a speed of $0.95\,v$, if v is the nominal speed of the aircraft. When changing direction on a waypoint, the heading of the line segment ”fastest point” changes as described. To check separation for two aircraft at time t, we compute the distance between the two line segments modeling the aircraft positions and compare it to the separation minima. In the vertical plane, we use a cylindrical modeling. Each aircraft has a mean altitude, a maximal altitude and a minimal altitude. To check if two aircraft are in conflict, the minimal altitude of the higher aircraft is compared to the maximal altitude of the lower aircraft.

It is possible to rewrite the equations of the previous model and to take into accounts the uncertainty of aircraft speeds. The equations become:

$$x_1 = v_1(1 + e_1)t - l_1$$

$$y_1 = 0$$

$$x_2 = \cos(\alpha)\,(v_2(1 + e_2)t - l_2)$$

$$y_2 = \sin(\alpha)\ (v_2(1+e_2)t - l_2)$$

Where e_1 and e_2 represent speed errors for aircraft 1 and 2. The development of this model has been done in another paper (see [Alliot1]). We are simply going to present the result which gives the additional percentage of conflict to detect and monitor compared to the lower bound found in the above section. We use the same notations: r stands for the ratio of speeds, α is the angle of incidence, D the standard separation, l_1 the distance to the crossing point and e the upper bound of the error (in percentage), such that e_1 and e_2 are always in the interval $[-e, e]$. Then the additional number of conflicts is given by:

$$\frac{2r\sin\alpha}{\sqrt{1+r^2-2r\cos\alpha}}\frac{l_1}{D}e$$

Let's do a numerical application. Both aircraft are flying at 360kts, so r=360/360=1, they are crossing with an angle of convergence α=90°, the bound on the speed error $e = 7\% = 0.07$, $D = 5\text{Nm}$ and we want to detect the conflict 5 minutes before the crossing point so $l_1 = 360 * \frac{5}{60} = 30\text{Nm}$. Then:

$$\frac{2r\sin\alpha}{\sqrt{1+r^2-2r\cos\alpha}}\frac{l_1}{D}e = \frac{2*1*1}{\sqrt{1+1-2*1*0}}*\frac{30}{5}*0.07 = 0.60$$

So we will detect and monitor 60% more conflict than the number of conflicts that really occur.

If we could download and use the information provided by the FMS then the advocates of MTCD tools think that the bound on the error would be extremely small and the number of conflicts wrongly detected would be considerably reduced.

This is only partially true. The FMS can provide a very accurate information on air speed. However, for detection purpose, its accuracy on ground speed depends on the accuracy of wind prediction. Of course, for resolution purpose, it would be possible to have the FMS enter a "closed loop" mode, where he would guarantee a given ETA on the crossing point (this is the idea behind the TCSA concept of SESAR). But it is impossible to use this kind of mode for each conflict detection, because we would have to compel aircraft to have an ETA for each crossing point, which would be much too complex and expensive. For conflict detection, even if the FMS provides perfect information on air speed and aircraft intentions, wind uncertainties have to be taken into account.

The influence of wind uncertainty on conflict detection

In this section we are going to build a mathematical model for conflict detection which takes into account wind errors.

In the rest of this paper, we are going to simplify further calculations by doing some approximations. First let's write:

$$\frac{v_2}{v_1} = r = 1 + \varepsilon$$

We will suppose that ε is small enough (aircraft speeds are quite similar) to discard second order quantities. Using the same notation than before, L/D becomes:

$$\frac{L}{D} = 2\frac{\sqrt{1 + r^2 - 2r\cos\alpha}}{\sin\alpha} = 2\frac{\sqrt{1 + (1+\varepsilon)^2 - 2(1+\varepsilon)\cos\alpha}}{\sin\alpha}$$

If we discard second order terms, and do some Taylor expansions in ε:

$$\frac{L}{D} = 2\frac{\sqrt{1 + (1+\varepsilon)^2 - 2(1+\varepsilon)\cos\alpha}}{\sin\alpha} = 2\frac{\sqrt{2(1+\varepsilon)(1-\cos\alpha)}}{\sin\alpha}$$
$$= 2\left(1 + \frac{\varepsilon}{2}\right)\sqrt{\frac{2(1-\cos\alpha)}{\sin(\alpha)^2}} = 2\left(1 + \frac{\varepsilon}{2}\right)\sqrt{\frac{2}{1+\cos\alpha}} = (1 + \frac{\varepsilon}{2})\frac{2}{\cos\frac{\alpha}{2}}$$

This gives us a much simpler formula. On figure 5, we have the new curve which is extremely similar to the previous one (here the x-axis represents the values of $\varepsilon = 1 - r$). Thus the approximations seem to be well-founded.

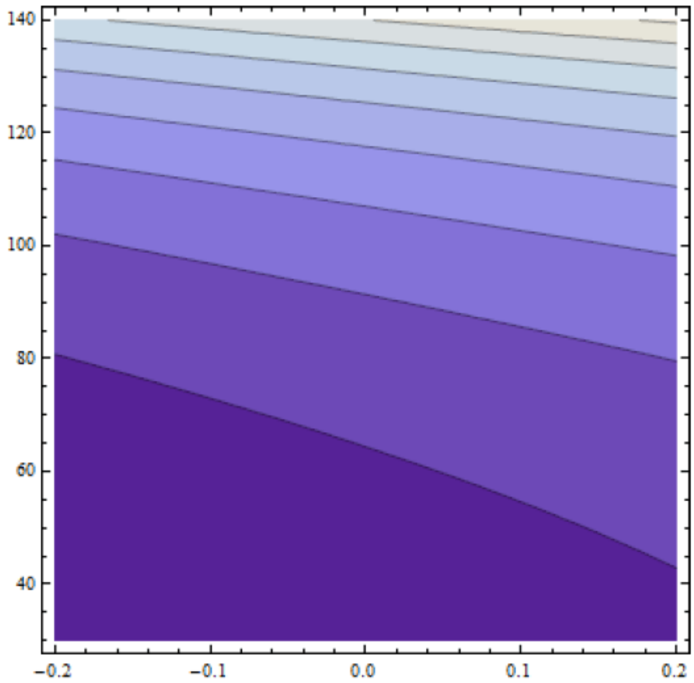


Figure 5

In the following part of the article, we will neglect without saying second order terms.

Now let’s suppose that the wind prediction is not perfect and that we have an unknown component of the wind defined by W (its module) and θ (its direction) (see figure 6).

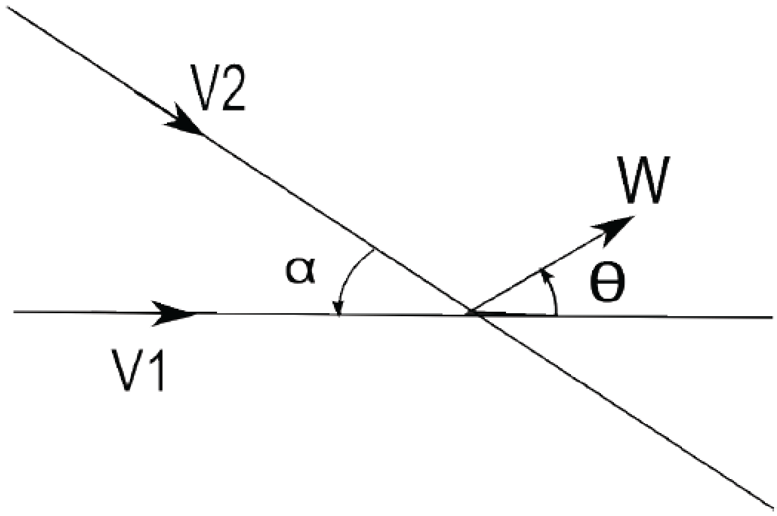


Figure 6

We suppose that aircraft automatically correct their heading by a drifting angle to maintain their course, and that they still maintain their air speed but not their ground speed (open loop hypothesis). Thus we have:

$$v_1' = v_1 + W\cos\theta$$

$$v_2' = v_2 + W\cos(\theta+\alpha)$$

We are going to use $w = \frac{W}{v_1}$ (w is supposed to be small) and $r' = \frac{v_2'}{v_1'} = 1+\varepsilon'$

$$r' = \frac{v_2'}{v_1'} = 1+\varepsilon' = \frac{v_2 + W\cos(\theta+\alpha)}{v_1 + W\cos\theta} = \frac{\frac{v_2}{v_1} + \frac{W}{v_1}\cos(\theta+\alpha)}{1+\frac{W}{v_1}\cos\theta}$$
$$= (1+\varepsilon + w\cos(\theta+\alpha))(1-w\cos\theta) = 1+\varepsilon + w(\cos(\theta+\alpha) - \cos\theta)$$
$$= 1+\varepsilon - 2w\sin(\theta+\frac{\alpha}{2})\sin\frac{\alpha}{2}$$

So now the two roots of the equation are (we use l instead of l_1 to simplify the notations):

$$r_1 = l(1+\varepsilon') + \frac{D}{\cos\left(\frac{\alpha}{2}\right)}\left(1+\frac{\varepsilon'}{2}\right)$$
$$= l\left(1+\varepsilon - 2w\sin\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\alpha}{2}\right) + \frac{D}{\cos\left(\frac{\alpha}{2}\right)}(1+\frac{\varepsilon}{2} - w\sin\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\alpha}{2})$$

$$r_2 = l(1+\varepsilon') - \frac{D}{\cos\left(\frac{\alpha}{2}\right)}\left(1+\frac{\varepsilon'}{2}\right)$$
$$= l\left(1+\varepsilon - 2w\sin\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\alpha}{2}\right) - \frac{D}{\cos\left(\frac{\alpha}{2}\right)}(1+\frac{\varepsilon}{2} - w\sin\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\alpha}{2})$$

If we suppose that we know an upper bound W_m (we will also use $w_m = \frac{W_m}{v_1}$) for the wind error module, we want to compute $\frac{L'}{D} = \frac{\max_{w,\theta} r_1 - \min_{w,\theta} r_2}{D}$ with w in $[0, w_m]$ and θ in [0,π]. It is easy to see that the maximum of $r_1$is at $w = w_m$ and $\theta = \frac{(3\pi-\alpha)}{2}$. Thus we have:

$$\max_{w,\theta} r_1 = l(1+\varepsilon) + \frac{D}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) + w_m \sin\frac{\alpha}{2}(2l + \frac{D}{\cos\frac{\alpha}{2}})$$

Finding the minimum of $r_2$is slightly more complex, and we first have to rewrite r_2:

$$\begin{aligned} r_2 &= l\left(1+\varepsilon - 2w\sin\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\alpha}{2}\right) - \frac{D}{\cos\left(\frac{\alpha}{2}\right)}\left(1+\frac{\varepsilon}{2} - w\sin\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\alpha}{2}\right) \\ &= l(1+\varepsilon) - \frac{D}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) - w\sin(\theta+\frac{\alpha}{2})\sin\frac{\alpha}{2}(2l - \frac{D}{\cos\frac{\alpha}{2}}) \end{aligned}$$

Thus the value of the minimum depends on the sign of $(2l - \frac{D}{\cos\frac{\alpha}{2}})$. There are two alternatives:

- If $l < \frac{D}{2\cos\frac{\alpha}{2}}$: this is when we are very close to the conflict point. Here we have the minimum of $r_2$for $w = w_m$ and $\theta = \frac{3\pi-\alpha}{2}$ and :

  $$\min_{w,\theta} r_2 = l(1+\varepsilon) - \frac{D}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) - w_m \sin\frac{\alpha}{2}(2l - \frac{D}{\cos\frac{\alpha}{2}})$$

  We still want to compute: $\frac{L'}{D} = \frac{\max_{w,\theta} r_1 - \min_{w,\theta} r_2}{D}$. Thus :

  $$\frac{L'}{D} = \frac{2}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) + 2w_m \tan\frac{\alpha}{2}$$

  Then the percentage of the increase of the number of conflicts is given by $\frac{L'/D}{L/D}$

  $$\frac{\frac{L'}{D}}{\frac{L}{D}} = 1 + w_m \sin\frac{\alpha}{2}\left(1-\frac{\varepsilon}{2}\right) = 1 + w_m \sin\frac{\alpha}{2}$$

  This first case is not very interesting as a simple numerical example will show; let's take D=5Nm and α=45°. Then this case applies if we detect conflict at a distance of roughly less than 3Nm to the crossing point which is obviously far too late.

- If $l > \frac{D}{2\cos\frac{\alpha}{2}}$: this is when we are far enough from the conflict point. We have the minimum for $w = w_m$ and $\theta = \frac{\pi-\alpha}{2}$ and :

  $$\min_{w,\theta} r_2 = l(1+\varepsilon) - \frac{D}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) + w_m \sin\frac{\alpha}{2}(2l - \frac{D}{\cos\frac{\alpha}{2}})$$

  Now

$$\frac{L'}{D} = \frac{2}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) + 4\frac{l}{D} w_m \sin\frac{\alpha}{2}$$

And (we suppress here again second order terms such as ε^2, w_m^2 or $w_m\varepsilon$ and do proper Taylor expansions):

$$\frac{\frac{L'}{D}}{\frac{L}{D}} = \frac{\frac{2}{\cos\frac{\alpha}{2}}\left(1+\frac{\varepsilon}{2}\right) + 4\frac{l}{D} w_m \sin\frac{\alpha}{2}}{\left(1+\frac{\varepsilon}{2}\right)\frac{2}{\cos\frac{\alpha}{2}}}$$

$$= 1 + 2\frac{l}{D} w_m \sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\left(1-\frac{\varepsilon}{2}\right)$$

$$= 1 + \frac{l}{D} w_m \sin\alpha\left(1-\frac{\varepsilon}{2}\right) = 1 + \frac{l}{D} w_m \sin\alpha$$

The main result of this paper is that the increase in the number of conflict is roughly:

$$\frac{l}{D} w_m \sin\alpha$$

Let's make a simple numerical application: aircraft is flying at $v_1 = 360$kts and the wind maximal error is known to be $W_m = 18$kts, so $w_m = \frac{W_m}{v_1} = \frac{18}{360} = 0.05$. If we want to detect conflicts 5 minutes before the crossing point, we have $l = 360 * 5/60 = 30$Nm. With a separation standard D=5 Nm, and a crossing angle= $\pi/2$:

$$\frac{l}{D} w_m \sin\alpha = \frac{30}{5}\,\frac{18}{360}\,\sin\frac{\pi}{2} = 6 * 0.05 * \ 1 = 0.3$$

So we will detect 30% conflicts more than the number of conflicts that will really occur, if we want to be sure to miss none of them. This number linearly increases with the anticipation. If we detect conflicts 15 minutes before the crossing point, we will detect 90% conflicts more, almost twice the actual number of conflicts, and this even if the MTCD tool is perfect (perfect FMS air trajectory prediction, no unexpected maneuvers by the pilots, etc).

Conclusion

SESAR promotes the "business owned" trajectory concept, the use of contract between air and ground to reduce the number of conflicts, and the development of ATC tools to facilitate the air traffic controller's tasks regarding conflict detection and resolution.

Enhanced on board navigation systems and data-link facilities offer new opportunities to develop these tools, and to further assist controllers in their detection and monitoring tasks. Thus, in the years to come, the role of the controller will probably shift from a mainly tactical work to a more strategic planning activity.

But even with a perfect collaboration between the board and the ground, we showed in this paper that future tools' efficiency will strongly rely on accurate wind prediction.

Thus a sustained effort is necessary to increase the quality of wind modeling and to reinforce the relationship between people working in both fields (meteorology and civil aviation) in order to promote a better understanding of the needs of both of them.

Very short bibliography

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